Let m_1 and m_2 be the mass of object 1 and the mass of object 2, respectively. Let v_{1i} and v_{1f} be the initial velocity of object 1 and the final velocity of object 1, respectively. Let v_{2i} and v_{2f} be the initial velocity of object 2 and the final velocity of object 2, respectively. Thus, the initial momentum (M_{1i}) and energy (E_{1i}) of object 1 are $M_{1i} = m_1 v_{1i}$ and $E_{1i} = \frac{1}{2} m_1 v_{1i}^2$. The final momentum and energy of object 1 are $M_{1f} = m_1 v_{1f}$ and $E_{1f} = \frac{1}{2} m_1 v_{1f}^2$, respectively. Finally, the initial momentum and energy and final momentum and energy of object 2 are $M_{2i} = m_1 v_{2i}$ and $E_{2i} = \frac{1}{2} m_1 v_{2i}^2$ and $M_{2f} = m_1 v_{2f}$ and $E_{2f} = \frac{1}{2} m_1 v_{2f}^2$, respectively. Thus, the total momentum and energy prior two the collision are $M_{1i} + M_{2i}$ and $E_{1i} + E_{2i}$, respectively and the total momentum and energy after the collision are $M_{1f} + M_{2f}$ and $E_{1f} + E_{2f}$, respectively. Since the total momentum before and after the collision is conserved, we have

$$\begin{aligned}
M_{1i} + M_{2i} &= M_{1f} + M_{2f} \\
m_1 v_{1i} + m_2 v_{2i} &= m_1 v_{1f} + m_2 v_{2f}
\end{aligned}$$

and since the total energy is conserved before and after the collision, we have

$$\begin{aligned}
E_{1i} + E_{2i} &= E_{1f} + E_{2f} \\
\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 &= \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \\
m_1 v_{1i}^2 + m_2 v_{2i}^2 &= m_1 v_{1f}^2 + m_2 v_{2f}^2.
\end{aligned}$$

Now we take the previous equation and solve for v_{2f}, which gives us

$$\begin{aligned}
m_1 v_{1i} + m_2 v_{2i} &= m_1 v_{1f} + m_2 v_{2f} \\
m_2 v_{2f} &= m_1 v_{1i} + m_2 v_{2i} - m_1 v_{1f} \\
v_{2f} &= \frac{m_1 v_{1i} + m_2 v_{2i} - m_1 v_{1f}}{m_2}.
\end{aligned}$$

Now we substitute this into the equation for energy and solve for v_{1f}. We get

$$\begin{aligned}
m_1 v_{1i}^2 + m_2 v_{2i}^2 &= m_1 v_{1f}^2 + m_2 v_{2f}^2 \\
m_1 v_{1i}^2 + m_2 v_{2i}^2 &= m_1 v_{1f}^2 + m_2 \left(\frac{m_1 v_{1i} + m_2 v_{2i} - m_1 v_{1f}}{m_2} \right)^2 \\
m_1 v_{1i}^2 + m_2 v_{2i}^2 &= m_1 v_{1f}^2 + m_2 \frac{m_1^2 v_{1i}^2 + 2 m_1 m_2 v_{1i} v_{2i} + m_2^2 v_{2i}^2 - 2 m_1^2 v_{1i} v_{1f} + m_1^2 v_{1f}^2 - 2 m_1 m_2 v_{2i} v_{1f}}{m_2^2} \\
m_1 m_2 v_{1i}^2 + m_2^2 v_{2i}^2 &= m_1 m_2 v_{1f}^2 + m_1^2 v_{1i}^2 + 2 m_1 m_2 v_{1i} v_{2i} + m_2^2 v_{2i}^2 - 2 m_1^2 v_{1i} v_{1f} + m_1^2 v_{1f}^2 - 2 m_1 m_2 v_{2i} v_{1f} \\
m_1 m_2 v_{1i}^2 &= m_1 m_2 v_{1f}^2 + m_1^2 v_{1i}^2 + 2 m_1 m_2 v_{1i} v_{2i} - 2 m_1^2 v_{1i} v_{1f} + m_1^2 v_{1f}^2 - 2 m_1 m_2 v_{2i} v_{1f} \\
m_1 m_2 v_{1f}^2 + m_1^2 v_{1f}^2 - 2 m_1^2 v_{1i} v_{1f} - 2 m_1 m_2 v_{2i} v_{1f} + m_1^2 v_{1i}^2 + 2 m_1 m_2 v_{1i} v_{2i} - m_1 m_2 v_{1i}^2 &= 0 \\
(m_1 m_2 + m_1^2) v_{1f}^2 - 2 m_1 (m_1 v_{1i} - m_2 v_{2i}) v_{1f} + m_1 v_{1i} (m_1 v_{1i} + 2 m_2 v_{2i} - m_2 v_{1i}) &= 0 \\
(m_1 + m_2) v_{1f}^2 - 2 (m_1 v_{1i} + m_2 v_{2i}) v_{1f} + v_{1i} (m_1 v_{1i} + 2 m_2 v_{2i} - m_2 v_{1i}) &= 0.
\end{aligned}$$

Now we are going to use the quadratic formula to solve for v_{1f}, which gives us

$$\begin{aligned}
v_{1f} &= \frac{-(-2(m_1 v_{1i} + m_2 v_{2i})) \pm \sqrt{(-2(m_1 v_{1i} + m_2 v_{2i}))^2 - 4(m_1 + m_2)(v_{1i}(m_1 v_{1i} + 2 m_2 v_{2i} - m_2 v_{1i}))}}{2(m_1 + m_2)} \\
&= \frac{2(m_1 v_{1i} + m_2 v_{2i}) \pm \sqrt{4(m_1 v_{1i} + m_2 v_{2i})^2 - 4(m_1 + m_2)(v_{1i}(m_1 v_{1i} + 2 m_2 v_{2i} - m_2 v_{1i}))}}{2(m_1 + m_2)} \\
&= \frac{m_1 v_{1i} + m_2 v_{2i} \pm \sqrt{(m_1 v_{1i} + m_2 v_{2i})^2 - (m_1 + m_2)(v_{1i}(m_1 v_{1i} + 2 m_2 v_{2i} - m_2 v_{1i}))}}{m_1 + m_2}.
\end{aligned}$$

Let us simplify what is under the radical. Doing so gives us

$$(m_1v_{1i} + m_2v_{2i})^2 - (m_1 + m_2)(v_{1i}(m_1v_{1i} + 2m_2v_{2i} - m_2v_{1i}))$$
$$m_1^2v_{1i}^2 + 2m_1m_2v_{1i}v_{2i} + m_2^2v_{2i}^2 - (m_1^2v_{1i}^2 + 2m_1m_2v_{1i}v_{2i} - m_1m_2v_{1i}^2 + m_1m_2v_{1i}^2 + 2m_2^2v_{1i}v_{2i} - m_2^2v_{1i}^2)$$
$$m_1^2v_{1i}^2 + 2m_1m_2v_{1i}v_{2i} + m_2^2v_{2i}^2 - m_1^2v_{1i}^2 - 2m_1m_2v_{1i}v_{2i} + m_1m_2v_{1i}^2 - m_1m_2v_{1i}^2 - 2m_2^2v_{1i}v_{2i} + m_2^2v_{1i}^2$$
$$m_2^2v_{2i}^2 - 2m_2^2v_{1i}v_{2i} + m_2^2v_{1i}^2$$
$$(m_2v_{2i} - m_2v_{1i})^2.$$

Now, we substitute this back into the previous equation and continue simplifying:

$$\begin{aligned} v_{1f} &= \frac{m_1v_{1i} + m_2v_{2i} \pm \sqrt{(m_2v_{2i} - m_2v_{1i})^2}}{m_1 + m_2} \\ &= \frac{m_1v_{1i} + m_2v_{2i} \pm m_2v_{2i} - m_2v_{1i}}{m_1 + m_2} \\ &= \frac{m_1v_{1i} + m_2v_{2i} + m_2v_{2i} - m_2v_{1i}}{m_1 + m_2}, \frac{m_1v_{1i} + m_2v_{2i} - m_2v_{2i} + m_2v_{1i}}{m_1 + m_2} \\ &= \frac{m_1v_{1i} + 2m_2v_{2i} - m_2v_{1i}}{m_1 + m_2}, \frac{m_1v_{1i} + m_2v_{1i}}{m_1 + m_2} \\ &= \frac{m_1v_{1i} + 2m_2v_{2i} - m_2v_{1i}}{m_1 + m_2}, \frac{v_{1i}(m_1 + m_2)}{m_1 + m_2} \\ &= \frac{m_1v_{1i} + 2m_2v_{2i} - m_2v_{1i}}{m_1 + m_2}, v_{1i} \end{aligned}$$

The solution $v_{1f} = v_{1i}$ would mean that the collision has had no effect on the collision, so we will discard this result and instead use $v_{1f} = \frac{m_1v_{1i}+2m_2v_{2i}-m_2v_{1i}}{m_1+m_2}$.

To solve for v_{2f}, we will use the save starting conditions. Now, we will solve for v_{1f} using the equation for momentum, which gives us

$$v_{1f} = \frac{m_1v_{1i} + m_2v_{2i} - m_2v_{2f}}{m_1}.$$

The calculation will mimic those given to solve for v_{1f}, so rather than show every step of the calculations, we will just focus on the main steps. Putting the previous equation into the energy equation and solving for v_{2f} gives us

$$m_1v_{1i}^2 + m_2v_{2i}^2 = m_1\left(\frac{m_1v_{1i} + m_2v_{2i} - m_2v_{2f}}{m_1}\right)^2 + m_2v_{2f}^2$$
$$(m_1 + m_2)v_{2f}^2 - 2(m_1v_{1i} + m_2v_{2i})v_{2f} + v_{2i}(m_2v_{2i} + 2m_1v_{1i} - m_1v_{2i}) = 0.$$

Using the quadratic formula to solve for v_{2f} gives us

$$\begin{aligned} v_{1f} &= \frac{-(-2(m_1v_{1i} + m_2v_{2i})) \pm \sqrt{(-2(m_1v_{1i} + m_2v_{2i}))^2 - 4(m_1 + m_2)(v_{2i}(m_2v_{2i} + 2m_1v_{1i} - m_1v_{2i}))}}{2(m_1 + m_2)} \\ &= \frac{m_2v_{2i} + 2m_1v_{1i} - m_1v_{2i}}{m_1 + m_2}, v_{2i}. \end{aligned}$$

Once again, we will discard $v_{2f} = v_{2i}$ as irrelevant and use $v_{2f} = \frac{m_2v_{2i}+2m_1v_{1i}-m_1v_{2i}}{m_1+m_2}$ as our equation for the final velocity of the second object.